

No-pumping theorem for non-Arrhenius rates

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The no-pumping theorem refers to a Markov system that holds the detailed balance, but is subject to a time-periodic external field. It states that the time-averaged probability currents nullify in the steady periodic (Floquet) state, provided that the Markov system holds the Arrhenius transition rates. This makes an analogy between features of steady periodic and equilibrium states, because in the latter situation all probability currents vanish explicitly. However, the assumption on the Arrhenius rates is fairly specific, and it need not be met in applications. Here a new mechanism is identified for the no-pumping theorem, which holds for symmetric time-periodic external fields and the so called destination rates. These rates are the ones that lead to the locally equilibrium form of the master equation, where dissipative effects are proportional to the difference between the actual probability and the equilibrium (Gibbsian) one. The mechanism also leads to an approximate no-pumping theorem for the Fokker-Planck rates that relate to the discrete-space Fokker-Planck equation.

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I. INTRODUCTION.

A wide range of systems appearing in physics, chemistry and biology can be modeled by Markov processes. Physically, Markov dynamics is the main tool for describing open systems (both quantum and classical) that interact with energy and/or particle reservoirs [1]. Hence it is at the core of non-equilibrium thermodynamics [2]. It is also the main tool for describing chemical reactions [3]. Among its biological applications one can mention conformational dynamics of biological molecules [4, 5], ion channel gating processes [6], dynamics of predation, epidemic processes, genetics of inbreeding [7] *etc.* Such applications are frequently developed within random walk models, e.g. chemotaxis, biological motions [8] *etc.*

Generically, a Markov dynamics with time-independent transition rates relaxes to a stationary state. For a single-temperature reservoir (equilibrium thermal bath) this stationary state amounts to the Gibbs distribution at the bath's temperature [1, 3]. The equilibrium nature of the bath is reflected in the detailed balance condition that ensures nullification of all probability currents in equilibrium [1, 3].

The concept of the stationary state is generalized, if the stochastic system is subject to an external time-periodic field [1, 14, 15]. The system still forgets its initial conditions and appears in a non-equilibrium, time-dependent state, whose probabilities oscillate with the same period as the external field. This is the content of the Floquet theorem (outlined below), and this motivates us to look at features of time-integrated (over the period of the field) probability currents from one state to another. Now the (no-pumping) theorem [9–13] states that probability currents nullify for an arbitrary time-periodic external field provided that the (time-dependent) transition rate $\rho_{i \leftarrow j}(t) > 0$ from state j to i holds the Arrhenius form

$$\rho_{i \leftarrow j}(t) = e^{B_{ij} + \beta E_j(t)}, \quad (1)$$

where $B_{ij} = B_{ji}$ refers to the time-independent transition state, $\beta = 1/(k_B T)$ is the inverse temperature, and $E_j(t)$ is the oscillating energy of the state j . Transitions from one state to another are induced by a thermal bath at temperature T , because if the bath is absent then due to energy conservation transitions between different states are also absent. Transition rates $\rho_{i \leftarrow j}(t)$ can be time-dependent solely due to an external field that acts on the system making its energies $E_j(t)$ time-dependent.

Thus the no-pumping theorem shows that the non-equilibrium, time-dependent state still holds an effectively equilibrium feature of nullifying (time-average) currents. (Whenever also B_{ij} in (1) are time-dependent, non-zero time-averaged currents are not excluded.) Hence the theorem fits naturally to the continuing effort of understanding the statistical mechanics of periodically driven systems using analogies with the equilibrium (i.e. time-independent) situation [15–18]. Recent works established several interesting relations between a driven system that hold the detailed balance condition and a similar system that is kept under constant (time-independent) non-equilibrium conditions [19]; in this context see also [20, 21].

Note that the same proof of the no-pumping theorem applies to rates more general than (1), but this generalization (though useful for its own sake) is achieved at the cost of violating the detailed balance condition [22]. I.e. formally the results of [22] refer to non-equilibrium baths.

The virtue of the no-pumping theorem is that it applies to all oscillating external fields. Its major drawback is that the Arrhenius form (1) does not hold in many important applications, where simultaneously the detailed balance is required. For example, the Metropolis rates (the main tool of the Monte-Carlo dynamics), hold (1) with $B_{ij} = -\beta \max[E_j(t), E_i(t)]$ (hence $\rho_{i \leftarrow j}(t) = \min[1, e^{\beta[E_j(t) - E_i(t)]]$), and B_{ij} cannot stay time-independent, if $E_i(t)$ and $E_j(t)$ are time-dependent. Further important examples of non-Arrhenius rates include Kramers rates that emerge out of diffusion in energy landscape [3] and corresponds in (1) to

$$\rho_{i \leftarrow j}(t) = e^{-\beta \delta_{ij} + \beta(E_j(t) - \max[E_i(t), E_j(t)])}, \quad (2)$$

where $\delta_{ij} = \delta_{ji}$ is energy barrier or activation energy that separates E_i and E_j . Another important example is the Fokker-Planck rates

$$\rho_{i \leftarrow j}(t) = e^{\beta[E_j(t) - E_i(t)]/2}, \quad (3)$$

which allow to match the discrete-space master equation for the Markov dynamics with the continuous-space Fokker-Planck equation [23]. For all these cases, the standard formulation of the no-pumping theorem would just allow non-zero time-averaged currents for a suitably chosen external field, i.e. the theorem is not very informative.

Here the aim is to extend the no-pumping theorem to rates different from (1) (the detailed balance is always assumed to hold).

First, it will be shown that the no-pumping theorem—time-integrated probability currents nullify—holds for the destination rates

$$\rho_{i \leftarrow j}(t) = e^{-\beta E_i(t)}, \quad (4)$$

under an additional sufficient condition that the external fields are (effectively) time-symmetric. The mechanism is more general since it nullifies the currents for certain non-symmetric external fields as well.

The destination rates (4) lead to the locally-equilibrium form of the master equation, which is driven by the difference between the actual probability and the equilibrium one; see the discussion after (14). There is a long and successful tradition of applying locally-equilibrium master equations in non-equilibrium physics. It was initiated via the model proposed in 1954 by Bhatnager, Gross and Krook [24–26], and since that time proved to be very useful [27]. In particular, the rates (4) were employed in [28] for describing the dynamics of a paradigmatic disordered statistical systems (the Random Energy Model), and found to be in agreement with experiments. Below I show that—in contrast to the Arrhenius rates (1)—the destination rates provide a reasonable approximation for other rates (e.g. the Fokker-Planck rate). Hence their experimental success is not accidental.

Second, it will be demonstrated numerically that the same mechanism that leads to the exact no-pumping theorem for the destination rates ensures an approximate validity of this theorem for the Fokker-Planck rate.

This work is organized as follows. The next section reviews the Markov master-equations and the Floquet theorem that is necessary for defining the no-pumping theorem. Section III is devoted to the analytical derivation of the no-pumping theorem for the destination rates and time-symmetric external fields. Here I also demonstrate that the time-symmetry is sufficient, but not necessary for the validity of the no-pumping theorem. Section IV studies the extent to which an approximate no-pumping theorem holds for more general rates (e.g. the Fokker-Planck rates). Section V briefly outlines the features of work invested to create the non-equilibrium state under study. The results are summarized in the last section.

II. MASTER-EQUATION AND THE FLOQUET THEOREM.

Consider a Markov master equation $[i, j = 1, \dots, n]$

$$\dot{p}_i \equiv dp_i/dt = \sum_j [\rho_{i \leftarrow j}(t)p_j - \rho_{j \leftarrow i}(t)p_i], \quad (5)$$

where $p_i(t)$ is the probability of the state i at time t , and $\rho_{i \leftarrow j}(t) > 0$ is the transition rate from j to i . It is assumed that for any fixed time t , there is the global detailed balance at inverse (time-independent) temperature β :

$$\rho_{i \leftarrow j}(t) e^{-\beta E_j(t)} = \rho_{j \leftarrow i}(t) e^{-\beta E_i(t)}. \quad (6)$$

Due to external field(s) acting on the system, the energies $E_i(t)$ are time-periodic functions with period τ :

$$E_i(t) = E_i(t + \tau). \quad (7)$$

The instantaneous probability flux from state j to state i is

$$J_{ij}(t) = \rho_{i \leftarrow j}(t)p_j(t) - \rho_{j \leftarrow i}(t)p_i(t). \quad (8)$$

Before specifying the external field, let me remind the Floquet theorem, which is necessary for defining the no-pumping theorem. Using the normalization of probabilities $\sum_{i=1}^n p_i = 1$, we write (5) as

$$\dot{P} = W(t)P(t) + b(t) \quad (9)$$

where $P(t) = [p_1(t), \dots, p_{n-1}(t)]$ and $b(t)$ are $(n-1) \times 1$ vectors and $W(t)$ is $(n-1) \times (n-1)$ matrix:

$$b_i = W_{in}, \quad W_{ij} = w_{ij} - w_{in}, \quad i, j = 1, \dots, n-1, \quad (10)$$

$$w_{ij} = \rho_{i \leftarrow j} - \delta_{ij} \sum_{k=1}^n \rho_{k \leftarrow j}, \quad i, j = 1, \dots, n. \quad (11)$$

The solution of (9) with initial condition $P(t_0)$ is

$$P(t) = A(t, t_0)P(t_0) + \int_{t_0}^t ds A(t, s)b(s), \quad A(t, s) \equiv \overleftarrow{e}^{\int_s^t du W(u)}, \quad (12)$$

where \overleftarrow{e} is time-ordered or chronological exponent. For $t \gg t_0$, the state $P(t_0)$ is forgotten, which is equivalent to $A(t, t_0)P(t_0) \rightarrow 0$. Taking $t_0 = -\infty$ in (12), and recalling that $W(t)$ and $b(t)$ are time-periodic with the same period,

we see from (12) that $P(t) = \int_{-\infty}^t ds A(t, s)b(s)$ is also time-periodic with the same period. This is the content of the Floquet theorem: for sufficiently long times, the stochastic system subject to time-periodic driving appears in a steady periodic state. This motivates us to characterize this state via time-averaged probability currents [cf. (8)]

$$\Phi_{ij} = \frac{1}{\tau} \int_a^{a+\tau} J_{ij}(t) dt, \quad (13)$$

where once the system is in its steady periodic (Floquet) state due to $a \gg t_0$, Φ_{ij} does not anymore depend on a . Below I shall employ $a = 0$ in the averaging. This implies that initial conditions are posed at much earlier time: $t_0 \rightarrow -\infty$.

Note that we do not consider cases, where the transition matrix describes an reducible chain. There the system does not generally forget its initial state.

III. NO-PUMPING THEOREM FOR A SINGLE FIELD AND DESTINATION RATES.

Using the normalization condition for probabilities $\sum_j p_j(t) = 1$ we obtain from (5) for the destination rates (4)

$$\dot{p}_i = e^{-\beta E_i(t)} - p_i(t)Z(t), \quad Z(t) \equiv \sum_j e^{-\beta E_j(t)}. \quad (14)$$

This is the main advantage of rates (4): the equations for the probabilities decouple from each other, making it convenient for studying systems with irregular distribution of energies [28]. Note that (14) can be written as $\dot{p}_i = -Z(t)[p_i(t) - \frac{e^{-\beta E_i(t)}}{Z(t)}]$ showing that the change \dot{p}_i of the probability is proportional to the difference between this probability and its equilibrium value $\frac{e^{-\beta E_i(t)}}{Z(t)}$. This makes connection between the studied destination rates and the Bhatnager, Gross and Krook kinetic equation [24–27].

Now (4, 8) imply

$$J_{ij} = \dot{p}_i p_j - \dot{p}_j p_i = p_j e^{-\beta E_i} - p_i e^{-\beta E_j}. \quad (15)$$

The no-pumping statement I propose is that for field (7) [plus additional symmetry conditions to be specified below], and for rates (4), it holds

$$\langle \dot{p}_i p_j \rangle \equiv \int_0^\tau dt \dot{p}_i(t) p_j(t) = 0, \quad (16)$$

thereby nullifying also the time-averaged current $\Phi_{ij} = 0$; see (15, 13). Note (16, 14) can be written as $\langle e^{-\beta E_i} p_j \rangle = \langle p_i p_j Z \rangle$. Hence the validity of (16) is obvious in the limiting case of very slow time-dependence, where the probabilities freeze to their Gibbsian (quasi-equilibrium) values: $p_i(t) = e^{-\beta E_i(t)} / Z(t)$.

To prove (16), we start from (14) and introduce there a new time-variable s

$$\frac{ds}{dt} = Z(t), \quad s = \int_0^t du Z(u). \quad (17)$$

Due to $Z(t) > 0$, the s -time relates to the t -time by a one-to-one mapping. Since $Z(t + \tau) = Z(t)$ [see (14, 7)], we get from (17):

$$s(t + \tau) = s(t) + \sigma, \quad \sigma = \int_0^\tau du Z(u). \quad (18)$$

Thus if $p_i(t)$ (in the Floquet regime) is τ -periodic, $p_i(t) = p_i(t + \tau)$, then $p_i(s)$ is σ -periodic:

$$p_i(s) = p_i(s + \sigma). \quad (19)$$

Note that the integral in (16) stays invariant under changing the time:

$$\int_0^\tau dt \dot{p}_i(t) p_j(t) = \int_0^\sigma ds \frac{dp_i(s)}{ds} p_j(s). \quad (20)$$

We get from (14)

$$\frac{dp_i(s)}{ds} = -p_i(s) + e^{-\beta E_i(s)} / Z(s). \quad (21)$$

We now introduce the Fourier-expansion for σ -periodic functions $g(s + \sigma) = g(s)$

$$g(s) = \sum_{n=-\infty}^{\infty} \hat{g}_n e^{\frac{2\pi i s n}{\sigma}}, \quad (22)$$

$$\hat{g}_n = \int_0^{\sigma} \frac{ds}{\sigma} g(s) e^{-\frac{2\pi i s n}{\sigma}} = \int_{-\sigma}^{\sigma} \frac{ds}{2\sigma} g(s) e^{-\frac{2\pi i s n}{\sigma}}. \quad (23)$$

and apply it to $p_i(s) \rightarrow \hat{p}_{i,n}$ and $e^{-\beta E_i(s)}/Z(s) \rightarrow \hat{\psi}_{i,n}$. Note that $\hat{g}_n^* = \hat{g}_{-n}$, since $g(s)$ is real. Eqs. (22, 21) imply

$$\hat{p}_{i,n} = \hat{\psi}_{i,n} \left[1 + \frac{2\pi i n}{\sigma} \right]^{-1}. \quad (24)$$

Using (24) we obtain for the integral in (20)

$$\int_0^{\sigma} ds \frac{dp_i(s)}{ds} p_j(s) = \sum_{n=-\infty}^{\infty} \frac{2\pi i n \hat{\psi}_{i,n} \hat{\psi}_{j,-n}}{1 + \left(\frac{2\pi n}{\sigma}\right)^2} = - \sum_{n=1}^{\infty} \frac{4\pi n \operatorname{Im}[\hat{\psi}_{i,n} \hat{\psi}_{j,-n}]}{1 + \left(\frac{2\pi n}{\sigma}\right)^2}, \quad (25)$$

where we employed the Fourier expansion of (21). If now $\hat{\psi}_{i,n} \hat{\psi}_{j,-n} = \hat{\psi}_{i,n} \hat{\psi}_{j,n}^*$ is real, the sum in (25) is zero. Thus the integrals in (25, 20, 16) nullify.

Let now $E_i(t)$ in (7) are even:

$$E_i(t) = E_i(-t). \quad (26)$$

Then $s(t)$ is an odd function of t [see (17,18)], and hence $E_i(s) = E_i(-s)$. Then $\hat{\psi}_{i,n}$ and $\hat{\psi}_{j,n}^*$ are real [see (23)], and the integral in (25, 20, 16) nullifies thereby proving the no-pumping theorem. A more general situation, when the same reasoning applies, and $\hat{\psi}_{i,n} \hat{\psi}_{j,-n}$ is real, takes place when $E_i(t)$ in (7) can be made even after a suitable time-shift γ which does not depend on i :

$$E_i(t - \gamma) = E_i(-t - \gamma). \quad (27)$$

This is because in the Floquet regime the origin of time can be chosen arbitrary. I stress that (27) gives only a sufficient condition for the validity of (16). The following example illustrates this fact. Let me take $i = 1, 2, 3 = n$ (three-level system), $\sigma = 1$ and define energies $E_i(s)$ so that the following relations hold

$$e^{-\beta E_i(s)}/Z(s) = c_i + d_i s + f_i s^2 \quad \text{for } 0 \leq s \leq 1, \quad (28)$$

while for $s > 1$ and $s < 0$, $e^{-\beta E_i(s)}/Z(s)$ is continued from (28) periodically with the period $\sigma = 1$. These functions are not continuous, but they can be considered as limits of continuous functions. This suffices for the sake of the present example.

In (28), c_i , d_i and f_i are constants, which should ensure the normalization and positivity of the probabilities $e^{-\beta E_i(s)}/Z(s)$. In particular, I choose $\sum_{i=1}^3 c_i = 1$ and $\sum_{i=1}^3 d_i = \sum_{i=1}^3 f_i = 0$ for normalization. Now generically (28) do not define any symmetric functions of s . However, we get

$$\hat{\psi}_{k,n} = \frac{f_k + i(d_k + f_k)n\pi}{2n^2\pi^2}, \quad (29)$$

$$\operatorname{Im}[\hat{\psi}_{i,n} \hat{\psi}_{j,-n}] = \frac{f_j d_i - f_i d_j}{4\pi^3 n^3}. \quad (30)$$

The nullification of all currents amounts to $\operatorname{Im}[\hat{\psi}_{i,n} \hat{\psi}_{j,-n}] = 0$ for all i and j . Generally, this requires three conditions $f_j d_i = f_i d_j$ to be imposed on f_i and d_i . But due to $\sum_{i=1}^3 d_i = \sum_{i=1}^3 f_i = 0$, it suffices to take a single condition $f_1 d_2 = f_2 d_1$. This ensures $f_j d_i = f_i d_j$ and thus nullifies all currents.

IV. APPROXIMATE NO-PUMPING

The above no-pumping theorem concerns the destination rates (4). It is not valid exactly for other interesting rates, e.g. Kramers (2) or Fokker-Planck (3); see Figs. 1–3.

Now Figs. 1–3 show numerical results, where the time-averaged current for a three-level system is compared for three different rates: destination (4), Kramers (2) and Fokker-Planck (3). Numerics was carried out for the following concrete form of $E_i(t)$:

$$E_i(t) = \varepsilon_i + a_i \cos\left(\frac{2\pi t}{\tau} + \varphi_i\right), \quad i = 1, 2, 3, \quad (31)$$

where ε_i , a_i and φ_i are constants. For (31) conditions (27) are satisfied e.g. for $\varphi_i = \varphi$ for all $i = 1, 2, 3$. This situation includes, e.g. the dipole coupling with an external, periodic electric field [1].

Figures 1–3 refer to different values of the time-period τ in (31). Figs. 1–3 demonstrate that under condition (27), the value of the time-averaged probability current nullifies exactly for the destination rates and it is approximately zero (with a good precision) for the Fokker-Planck rates (denoted as F–P in Figs. 1–3). For the Kramers rates the situation is different: it also predicts an approximately zero time-averaged probability current, but only for a sufficiently large τ ; see Fig. 1.

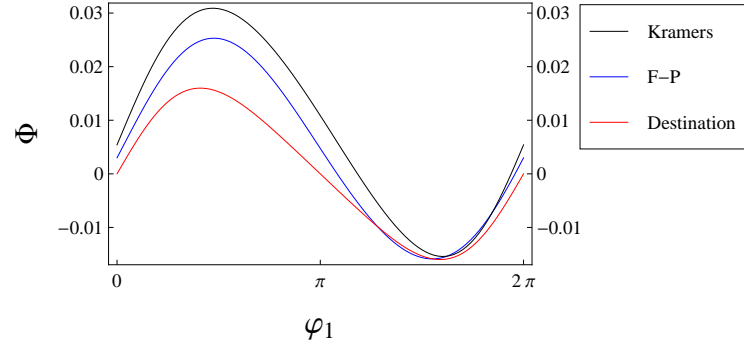


FIG. 1. Time-averaged current $\Phi = \Phi_{12} = \Phi_{23} = \Phi_{31}$ given by (13) for a three-level system ($n = 3$) and $\beta = 1$ versus the parameter φ_1 for Kramers, Fokker-Planck (F–P) and destination rates; see (31). $E_i(t)$ are given by (31), where $\tau = 3$. Other parameters in (31): $\varphi_2 = \varphi_3 = 0$, and $\varepsilon_1 = \frac{1}{3}$, $a_1 = 1$, $\varepsilon_2 = \frac{2}{3}$, $a_2 = 2$, $\varepsilon_3 = 1$, $a_3 = 3$. For Kramers rates $\delta_{ij} = 1$ in (2). Hence for $\varphi_1 = 0$ or $\varphi_1 = 2\pi$, the external field satisfies (27) and holds the no-pumping theorem $\Phi = 0$ for the destination rates, as seen on the figure. If (27) holds, $\Phi \approx 0$ for the Kramers rates (1, 2) and the Fokker-Planck rates (1, 3).

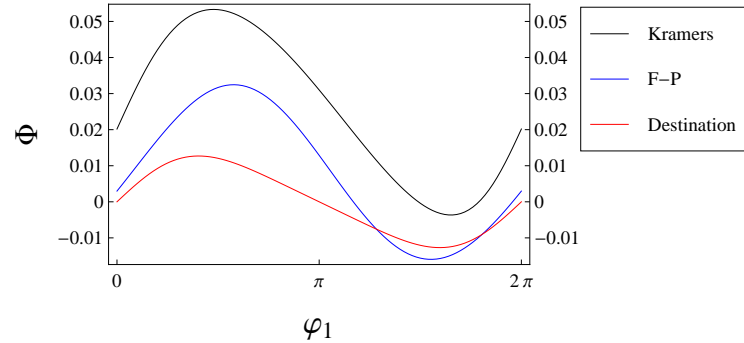


FIG. 2. The same as in Fig. 1, but with $\tau = 1$, i.e. the external fields change faster than in Fig. 1, where $\tau = 3$. For this range of parameters the Fokker-Planck rates hold an approximate no-pumping theorem, while the Kramers rates do not.

Fig. 4 gives an example of a situation, where (for all studied rates) the time-averaged currents are sizable, since conditions (27) do not hold.

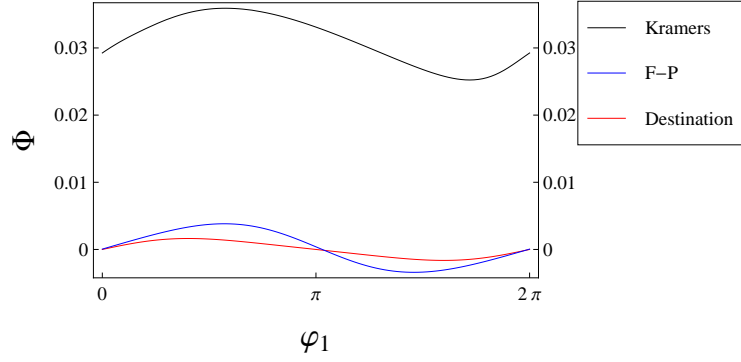


FIG. 3. The same as in Fig. 1, but with $\tau = 0.1$; cf. Fig. 2. The external fields change faster than in Fig. 1 and in Fig. 3. The no-pumping theorem approximately holds for the Fokker-Planck (FP) rates.

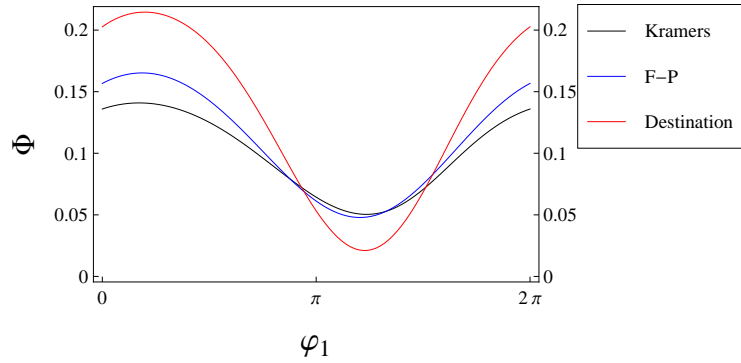


FIG. 4. The same as in Fig. 1, but $\varphi_2 = \pi, \varphi_3 = \frac{3\pi}{2}$. In this example conditions (27) do not hold and the probability currents are sizable for all studied rates.

Note that all above numerical examples did not refer to high temperatures. Clearly, probability currents generally nullify for large temperatures, but one can identify a regime, where the *instantaneous* time-dependent currents are still sizable, though their time-averages are practically zero. This is shown in Fig. 5, where the ratio between instantaneous and averaged currents amounts to $\sim 10^{-3}$. This high-temperature version of the no-pumping theorem holds for all studied rates and it does not need conditions (27).

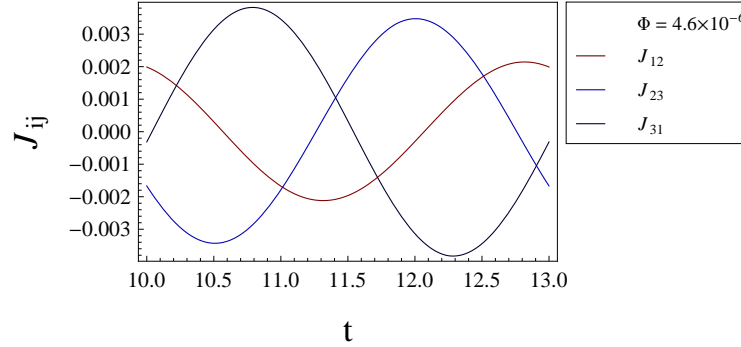


FIG. 5. Instantaneous probability currents $J_{ij}(t)$ given by (8) for $\beta = 0.01$ and Kramers rates (1, 2). In (31) I took: $E_i(t) = -\frac{i}{2} + \frac{i}{2} \cos\left(\frac{2\pi t}{3} + \frac{\pi i}{2}\right)$, and for barriers: $\delta_{ij} = 1$. It is seen that J_{ij} are much larger than their time-average $\Phi = \Phi_{12} = \Phi_{23} = \Phi_{31}$.

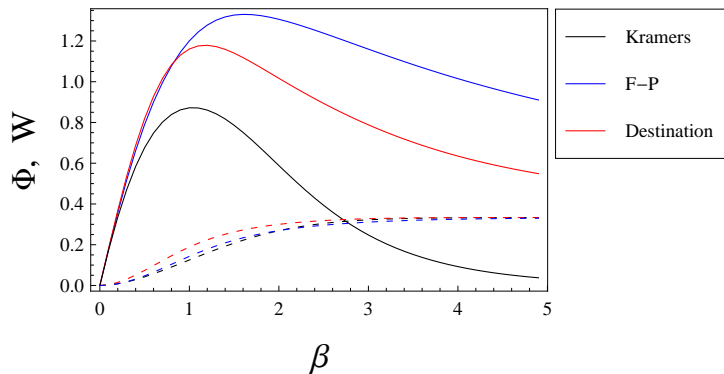


FIG. 6. Time-averaged current $\Phi = \Phi_{12} = \Phi_{23} = \Phi_{31}$ (dashed curves) given by (13) and W given by (34) versus the inverse temperature β and for various rates. In (31) I took: $E_i(t) = \frac{i}{3} + i \cos(\frac{2\pi t}{3} + \frac{i\pi}{2})$.

V. WORK.

To keep the system in the non-equilibrium state, the external field dissipates work into the thermal bath. Now the work relates to energy (and not probability) currents through the system. Hence it is important to study it in the context of the no-pumping theorem.

The rate of work can be calculated via the standard formula

$$\frac{dW}{dt} = \sum_i p_i(t) \dot{E}_i. \quad (32)$$

The positivity of the time-averaged work W is deduced from the positivity of the entropy production (see e.g. [29] for this concept, its physical meaning is clarified below)

$$S_e(t) = \frac{1}{2} \sum_{ik} (w_{ki} p_i - w_{ik} p_k) \ln \frac{w_{ki} p_i}{w_{ik} p_k} \geq 0, \quad (33)$$

where w_{ik} is defined in (11). Now writing as $\ln \frac{w_{ki} p_i}{w_{ik} p_k} = \ln \frac{w_{ki}}{w_{ik}} + \ln \frac{p_i}{p_k}$, we note that the second term amounts in (33) to $-\frac{d}{dt} \sum_i p_i \ln p_i$ and thus disappears after the time-averaging due to the Floquet theorem. Using (6) and again the Floquet theorem we obtain from the first term the positivity of the time-averaged work:

$$W = \int_0^\tau dt \sum_i p_i(t) \dot{E}_i = - \int_0^\tau dt \sum_i \dot{p}_i(t) E_i = T \int_0^\tau dt S_e(t) \geq 0. \quad (34)$$

Applying the Clausius inequality to the bath—recall that W turns to the heat Q received by the equilibrium thermal bath at temperature T , and then Q/T is smaller or equal to the bath entropy increase—it is seen that $\int_0^\tau dt S_e(t)$ gives a lower bound for the bath entropy increase per cycle.

Note from Fig. 6 that the average work decays to zero both for high and low temperatures. There is no no-pumping (i.e. no-work-dissipation) theorem for it.

VI. SUMMARY

It was shown that there is a mechanism by which the no-pumping theorem—time-averaged probability currents nullify in the Floquet regime—holds for the destination transition rates (4) (which hold the detailed balance condition). A sufficient condition for this validity is that the external time-periodic fields acting on the stochastic system hold the time-symmetry (27). Similar time-symmetry conditions (together with the space symmetry of the external potential) govern the current-generation regimes of various ratchet models; see [30] for a recent review. It should be interesting to understand in more detail possible relations between the no-pumping theorem and the (no) current-generation in ratchets; this is left for future work.

In the regime, where the no-pumping theorem holds exactly for the destination rates, there is an approximate no-pumping theorem that holds for the physically pertinent Fokker-Planck rates.

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